Research Interests:

My research focuses on imaging complex structures, primarily within the Earth, using modern mathematical techniques. Starting from the underlying mathematical models, I am interested in exploring connections between techniques used in different fields; specifically, the exchange of ideas between exploration and global seismology. In these two areas, as well as many others, multiply scattered energy is discarded as noise. I hope to pursue the goal of finding ways to instead exploit the information contained within multiply scattered wavefields. Thus far, my research has spanned a range of topics, from theoretical and computational developments to laboratory experiments; this gives me a broad perspective from which to look for new problems and new ideas for established problems.

Completed Research:

As a PhD student, I worked on three different research projects encompassing theoretical and computational work, as well as laboratory experiments:

Data Continuation: Data continuation is the computation of data missing in a field acquisition, using the data that were recorded. We developed a general theory to continue data, making few simplifying assumptions about the data themselves. Such techniques are needed in exploration seismology to construct a data set appropriate for current methods of imaging. The theoretical development involves the composition of Fourier Integral Operators (a generalization of the Fourier transform) to construct a single operator that maps directly from the input data to an output data set, which is sampled differently than the input. We were able to show, under minimal conditions on the underlying Earth structure, that this composite operator is again a Fourier Integral Operator, meaning that no spurious reflections are introduced in the data during the continuation process. On the practical side, we developed an algorithm to test these ideas on synthetic seismic data. We were able to reconstruct missing data, using information about the underlying Earth structure, even when the missing portion of the data contained caustics.

Characterizing Mesoscopic Phenomena in Rocks: The goal of this project was to characterize the mesoscopic properties of coarse-grained rocks, using the latest non-contacting ultrasonic measurement techniques. We first collected data on three samples with different grain sizes, ranging from homogeneous to grains with diameter about three times the wavelength. We were then able to fit these data with a radiative transfer model to infer the mean-free path, a measure of the scattering strength of the coarsest-grain sample. To further explore the influence of scattering on the wavefield, we tracked the transition of the wavefield from ballistic propagation (energy traveling as a packet) to equipartitioning, in which equal amounts of energy propagate in all directions. (Equipartitioning is a mesoscopic analog of thermal equilibrium.) The time it takes to make this transition is dependent on the scattering strength of the sample. We followed this transition using the result that the correlation of fields recorded at two different points gives a weighted sum of the causal (spike at positive time) and anti-causal (spike at negative time) Green's functions for waves traveling from one receiver to the other. In the ballistic regime, the causal Green's function is much stronger, whereas in the equipartioned regime both have equal weight. The symmetry of different wave types may also provide a method to discriminate scattering of a particular wave type from the bulk scattering.

Inverse Multiple Scattering: Internal multiples violate one of the fundamental assumptions of seismic processing, that all data have scattered only once in the subsurface before being recorded at the surface. Because of this, internal multiples are responsible for artifacts in seismic images. The majority of approaches to attenuating internal multiples work in the data domain; the multiples are estimated in the data and then subtracted before an image is formed. We developed a technique to estimate these artifacts directly in the image, creating an algorithm that estimates the multiples as an addition to a standard migration algorithm. The theoretical development involves the construction of two scattering series, one to model data given the underlying Earth structure and the other to estimate the underlying Earth structure from recorded data. Results on both synthetic and real exploration seismic data are promising.

Ongoing Research:

Multiples in radar data: Radar range data is used in both Earth imaging and identification of objects from a distance. As with seismic imaging, radar imaging algorithms generally assume that all energy scatters only once, which is not always the case. Using techniques similar to those applied in exploration seismology, I have been working on a theoretical framework and algorithm to estimate the multiple bounces in radar data. Preliminary results suggest that this technique allows the time and shape of multiply scattered waves to be estimated in radar data. Such information can be used to attenuate these waves before forming a radar image, thus creating data that satisfy the single scattering assumption. Alternatively, the estimated multiples could be used to estimate artifacts directly in the radar image. Both approaches would result in better images.

Ultrasound vibro-acoustography: Ultrasound vibro-acoustography is a method used to create high resolution images without the speckle caused by scattering from small inhomogeneities. The method exploits an effect from second-order corrections to the wave equation, derived directly from the Euler equations. So far, most of the work on this imaging modality has been experimental, and there are many theoretical and modeling problems that have not yet been solved. This project has two goals. The first is to be able to model the imaging process, so that we can suggest changes to the experiment to improve the images. The second goal is to help the experimentalists develop robust tools for solving the inverse problem of determining tissue properties from the image. Few attempts have been made to model these experiments because of the cost associated with traditional modeling methods. We are currently working to adapt fast migration methods from seismology to alleviate this problem.

Future Directions:

In general, I will continue to use techniques of modern mathematics to solve imaging problems in the Earth sciences. Having worked as both a theoretician and an experimentalist, I see the importance of these two approaches working in concert. I intend to develop collaborations that allow my research to grow along with developments in laboratory and field experiments.

Most techniques, including the one developed as part of my thesis research, treat multiply scattered energy as noise to be attenuated rather than as signal carrying information about the Earth's structure. I will work to integrate my mathematical understanding of multiples on the exploration scale and my knowledge of the extreme forms of multiple scattering that we see in the lab to lead the way in upgrading the status of multiples from noise to signal. Since they spend more time in a particular region of the Earth, multiples are more sensitive to that region than singly scattered waves (primaries) are. Thus, they should be useful as part of a tomography procedure, or even to improve constraints on the singular structures from which they scatter. This work has obvious applications in exploration seismology, where velocity estimation near multiple-generating salt structures is a problem of current interest. On the global scale, multiples may be able to fill in some of the gaps in recording caused by the oceans; this would require significant theoretical developments beyond what is done in exploration but which may be possible in the future. In the near surface, multiples can be particularly troublesome due to difficulties in estimating the velocity structure; this is a topic of current interest to researchers working with ground penetrating radar data.

In the spirit of applying modern mathematical techniques to analyze Earth science problems, I would like to investigate the transition of a wavefield from ballistic propagation to equipartitioning through mathematical, rather than experimental, means. Specifically, I would like be able to quantify the point at which the energy propagating through a heterogeneous medium begins to propagate with equal strength in all directions. The time for this transition to take place is related to the scattering strength of the material; understanding it will help to characterize the heterogeneity of the sample. It is not immediately clear what tools are necessary to track this transition. While Fourier methods give some estimate of direction, they are not local and, thus, cannot tell us how the energy at one location is moving. Wavelets seem a likely candidate, but for multi-dimensional data it is not clear what extension of one-dimensional wavelets is optimal for this task.

I believe that exploration seismology and global seismology have a lot to teach one another, and I would like to be involved in this ongoing exchange of ideas. For example, the global seismology community is beginning to collect data sets on which some of the high-resolution imaging techniques of exploration could be applied. This is the area that I am most interested in investigating as a member of an Earth science department. I am only beginning to learn the questions, data limitations, and common techniques of global seismology. I find these problems fascinating, and the signals seen in field records are as impressive as those recorded in a laboratory in terms of the identification of small signals within a complicated waveform. From the other side, the oil industry is becoming more interested in extracting information about poorly resolved structures with limited data. As someone whose expertise is currently focussed in exploration seismology, I would hope to learn more about the techniques used in global seismology, which I think is ahead of exploration in this area.

Although I am enjoying the opportunity, at the Institute for Mathematics and its Applications (IMA), to work in different disciplines, I see my focus returning to Earth science problems quickly upon joining an Earth science department. I am learning a lot at the IMA about different types of imaging, and I think that geophysics could benefit from some of these techniques. I would like to pursue some of these connections by improving my understanding of the mathematics underlying both the problems of geophysics and the imaging techniques used in other fields.